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THE MANY USES OF CHIRAL EFFECTIVE THEORIES

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Abstract

I review basic concepts of chiral effective field theories guided by an historical perspective: from the first ideas to the merging with other effective frameworks, and to the interplay with lattice field theory. The impact of recent theoretical developments on phenomenological predictions is reviewed with attention for rare decays, and charm physics. I conclude with a critical look at future applications.

1   A retrospective

Effective field theories are the protagonists of our modern view of quantum field theory. The idea that any sensible theory is a priori valid only on a limited interval of energies, else distances, became more and more accepted during the last decades. Any such theory carries a dependence on a particular

high energy scale, *the ultraviolet cutoff* which determines its range of validity. It contains the low energy or large distance behaviour of a more fundamental theory. Only the ultimate fundamental theory, if any, must be valid at all energies or distances. This broadened view led us to abandon the concept of renormalizability in a strict sense as the necessary requisite for a theory to be an acceptable theory. Effective field theories are by now one of the main theoretical instruments for exploring a large set of particle physics phenomena, from the very low-energy strong interactions to the candidate models for the physics beyond the electroweak symmetry breaking scale. In this writeup I will care for concepts more than numbers, and make use of what an historical perspective can teach us.

Effective field theories (EFT) started as *phenomenological lagrangians*, aimed at describing the dynamics of strongly interacting matter, mesons and baryons, at low energy. In general, they were aimed at describing any system where the dynamics is governed by a given internal symmetry and its spontaneous breaking. The original works date in the 1960's, mainly by Schwinger, Cronin, Weinberg [1, 2, 3], and the works by Callan, Coleman, Wess, Zumino [4, 5]. The structure of phenomenological lagrangians was purely based on symmetries and much inherited from current algebra; here, the low energy strong interactions could be described by a formulation alternative to Quantum Chromodynamics (QCD)[1], the theory with quarks and gluons degrees of freedom.

Chiral Perturbation theory (ChPT) was formulated more than a decade later by Gasser and Leutwyler in two by now well known papers [7, 8] in 1984 and 1985. ChPT is the descendant of phenomenological lagrangians. It is a particular example of a non-decoupling effective theory. Its fundamental symmetry is the chiral symmetry, with group $SU(2)_L \times SU(2)_R$ or $SU(3)_L \times SU(3)_R$[2], spontaneaously broken down to its diagonal subgroup. The derivation of the ChPT lagrangian and properties by a path integral formulation [7] clarifies its field theoretical connection to QCD. Somewhat more recently Heavy Quark effective theory (HQET) was introduced as a good theoretical approximation

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[1] Phenomenological lagrangians evolved together with the concept of *quarks* degrees of freedom [6] and the description of strong interactions with a non abelian gauge theory

[2] I will omit the subscripts L, R in the following.

to describe the dynamics of systems with one heavy quark [9]. It is the merging of these two formulations that gave rise to new types of effective field theories, namely the Heavy Baryon ChPT [10] (HBChPT) for describing the interactions amongst baryons and light mesons, and the Heavy-Light Meson ChPT [11] (HLChPT) for describing the dynamics of bound meson systems such as D, D_s, B, and B_s.
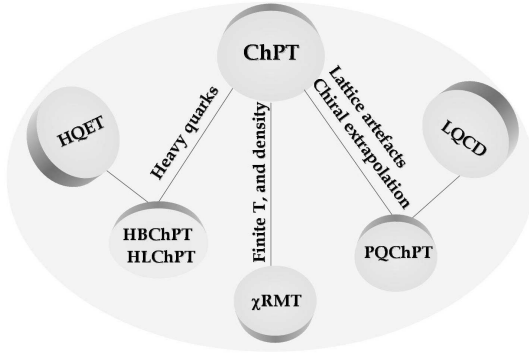

Figure 1: *The merging of ChPT with HQET and lattice QCD gives rise to new effective field theoretical descriptions of physical phenomena like baryon and heavy-light meson physics, field theory at finite volume, on discretized space-time and in the (partially) quenched limit (PQChPT). Another branch in the figure points at the particularly interesting realization of Chiral Random Matrix Theory [14] (χRMT) recently proved to be equivalent to ChPT [15], a powerful tool to explore QCD at finite temperature and nonzero chemical potentials.*

Moving forward in time we encounter a fruitful interplay of this wide class of EFT descriptions with the lattice formulation of field theories and in particular of strong interactions (referred to as Lattice QCD (LQCD)). During the last years it has become clear how the complementary use of both approaches is extremely useful to understand non perturbative aspects of a field theory, possibly gaining insight into the way for an exact solution; one exciting example is the attempt at extending the AdS/CFT conjecture [12] to AdS/QCD [13].

In the following sections I review the basic principles of phenomenological lagrangians, their descendants, and discuss a few topics in the phenomenolgy of

hadron interactions, where the role of EFT and LQCD is and will be especially relevant. I conclude with some thoughts on possible future developments and applications.

2   The Theory

The formalism of phenomenological lagrangians was mainly motivated by the necessity of describing the interactions of phenomenological fields, like the pions, whose appearance is due to the spontaneous breaking of an internal global symmetry. The mathematical problem is equivalent to that of finding all nonlinear realizations of a (compact, connected, semisimple) Lie group which become linear when restricted to a given subgroup [4]. The following problem is the one of constructing nonlinear lagrangian densities which are invariant under the nonlinear field transformations [5].

Consider the chiral group $G = SU(N) \times SU(N)$ which is spontaneously broken down to the diagonal (parity-conserving) subgroup $H = SU(N)_V$. The pattern of symmetry breaking is $SU(N)_L \times SU(N)_R \to SU(N)_V$ with $N = 2, 3$ flavours. Of the total number of generators of G, there will be $N^2 - 1$ "exact" generators of the subgroup H, and $N^2 - 1$ "broken" generators of the residual subgroup. Fields are associated to the generators of G, and pions, the Goldstone bosons of the spontaneaously broken chiral symmetry, are associated to its broken generators. Current algebra was implying that to correctly describe pion interactions it was necessary to eliminate all non derivative couplings from the lagrangian. The mathematical solution and the construction of the correct lagrangian for pions was reached in two ways: from the old σ-model, by performing [2] a chiral rotation of the four-dimensional field (σ, π) of $SU(2)$, which eliminates the non derivative coupling of σ and π and replaces it with a nonlinear derivative coupling of the chiral rotation vector, identified as the new pion field and transforming as a nonlinear realization of G.[3] The second more elegant way [3, 4, 5] was to directly postulate the nonlinear transformation properties of the pion fields and to construct a G (chiral) invariant lagrangian.

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[3]The fact that one can limit the field content to a pion triplet and does not need to add a scalar field is due to the existence of a three-dimensional non linear realization of $SU(2) \times SU(2)$ while there is no three-dimensional linear representation.

The recipe [5] for such a lagrangian amounts to

$$\mathcal{L} = cTr\left[\partial_\mu\left(e^{i\pi\cdot T}\right)\partial^\mu\left(e^{-i\pi\cdot T}\right)\right] , \qquad (1)$$

where $\pi\cdot T = \sum_{i=1}^{N^2-1}\pi_i\,T_i$, with T_i the broken generators and π_i the associated pion fields. The coupling constant c is proportional to the scale of the symmetry breaking. This lagrangian describes self-interactions of the phenomenological fields π, and it is nonlinear in the fields: its exponential dependence generates infinitely many interaction terms. Another peculiarity is that it contains only derivative type interactions, which means that at low energies the fields are weakly interacting. In a paper of 1979 [16] entitled "Phenomenological Lagrangians", Weinberg constructed the renormalization group relations amongst the divergent structures appearing in the loop expansion of the theory. These relations clarify in which sense the theory is nonrenormalizable.

The formulation of ChPT appeared in two seminal papers by Gasser and Leutwyler; in the first [7] the $SU(2)$ flavour theory is derived, and in the second [8] the theory is extended to SU(3) to include the heavier strange quark. The lagrangian

$$\mathcal{L}_2 \;=\; \frac{f_\pi^2}{4}Tr\left(D_\mu\Sigma D^\mu\Sigma^\dagger\right) \;+\; \frac{f_\pi^2}{4}Tr\left(\chi^\dagger\Sigma + \chi\Sigma^\dagger\right)$$

is the first order contribution to an expansion in powers of the small energies of the fields $\Sigma = e^{\frac{2i}{f_\pi}\Phi\cdot T}$ and the light quark masses, which are invariantly introduced through the scalar spurion field $\chi = 2B_0\mathcal{M} + \ldots$, with $M = diag(m_u, m_d, m_s)$ and B_0 the parameter proportional to the scalar quark condensate. Covariant derivatives are defined to contain external vector and axial spurion fields $D_\mu\Sigma = \partial_\mu\Sigma - i(v_\mu + a_\mu)\Sigma + i\Sigma(v_\mu - a_\mu)$.

ChPT is the effective description of a strongly coupled theory, which is low energy QCD. Its expansion in powers of small momenta and light quark masses

$$\mathcal{L} \;=\; \mathcal{L}_2 \;+\; \frac{1}{\Lambda_\chi^2}\mathcal{L}_4 \;+\; \ldots \qquad\qquad p^2 \sim M_\pi^2 \sim m_q$$

has a predictive power which is dictated by the numerical value of its ultraviolet cutoff, by construction the scale of spontaneous chiral symmetry breaking $\Lambda_\chi \propto f_\pi$, the pion decay constant. The equality $\Lambda_\chi \simeq 4\pi f_\pi \simeq 1\,GeV^4$ guarantees a

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[4] It is suggested by the numerical behaviour of the loop expansion, and not derived from first principle considerations.

good predictivity for energies well below 1 GeV.

Flavour physics involving dominant contributions from low energy strong interactions can be explored with ChPT: the SU(2) case completely describes pion physics and the physics of isospin breaking for $m_u \neq m_d$. The SU(3) case describes kaon physics, with $m_u, m_d \ll m_s$ and provided the kaon mass $M_K \ll \Lambda_\chi$. ChPT plays an essential role in the calculation of QCD induced corrections to weak decays of light mesons. Golden channels are certainly the nonleptonic kaon decays, source of the $\Delta I = 1/2$ rule and probe of indirect CP violation through ε'/ε, and rare kaon decays, useful to constrain sources of new physics beyond the standard model. There are special cases where large corrections to $SU(3)$ processes are purely $SU(2)$ effects, as it is for final-state-interactions in $K \to \pi\pi$ decays [17].
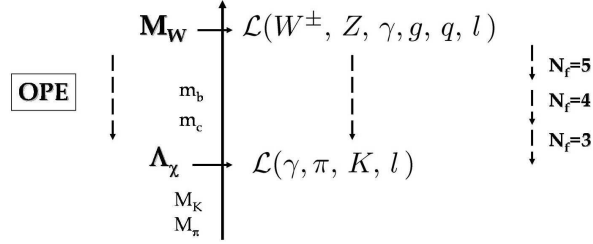

Figure 2: *Flow diagram of the unified EFT description of weak and strong interactions. N_f is the number of active flavours in a given theory. By decreasing energies, the top quark and W boson, the bottom quark, the charm quark are subsequently "integrated out". Λ_χ is the chiral symmetry breaking scale at which the nonperturbative matching of ChPT with QCD is performed.*

The unified description of weak and strong interactions as effective field theories is the perfect example of the two ways in which an EFT can be realized. Weak interactions identify with a weakly coupled theory, where decoupling (of massive modes) takes place and perturbative matching can be performed. In a sequence of decreasing energies (see fig.2), starting at the mass of the W boson, the renormalization group equations and Operator Product expansion (OPE) run the theory to lower energies. Massive modes "decouple" from the theory at

each threshold[5] and give rise to a new EFT realization. The matching of two theories above and below the matching scale is genuinely perturbative. Strong interactions identify with a strongly coupled theory, which does not decouple. Hence, an EFT realization will arise via a nonperturbative matching with the fundamental theory; at the chiral symmetry breaking scale Λ_χ, quarks leave the ground to pions and kaons, through the nonperturbative matching of ChPT with QCD.

3   Its descendants

As symmetries are the foundations of any effective field theory description, we can look for extensions of ChPt through its merging with the realization of additional symmetries and their breaking. A particularly fruitful example is the merging of ChPT with the Heavy Quark Effective Theory (HQET) formulated around 1990 [9]. The additional symmetry is in this case the one recovered in the limit of infinitely heavy fermions: the heavy quark spin symmetry. The descendants of ChPT are nowadays proliferating, especially after it was realized how the interplay of ChPT with lattice QCD can be an invaluable guidance to the theoretical interpretation and improvement of lattice calculations; an EFT description can be formulated for each purpose, describing the dependence upon the volume, the lattice spacing, the fermion masses, mimicking and parameterizing the behaviour of a specific lattice formulation. With the caveat of a limited energy-range of validity, it offers a rigorous theoretical background to interpret physical phenomena on the base of symmetries and group theoretical properties.

3.1   The merging with Heavy Quark Effective Theory

The Dirac theory for spin $1/2$ fermions can be reshaped in the limit of an infinitely heavy quark. Additional symmetries are restored in this limit, namely the heavy quark spin symmetry [9]. The merging of HQET with ChPT, gave birth to the effective description of baryon interactions, known as Heavy Baryon ChPT (HBChPT) [10]. When baryon number is conserved and taking into

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[5]The way decoupling manifests depends on the renormalization scheme used. Smooth decoupling does not arise in the typically used $\overline{MS}$ scheme where the decoupling consists of "integrating out" the corresponding massive particle.

account that $m_B \simeq \Lambda_\chi$, we can factor out the baryon mass from the total momentum and expand in $1/m_B$. The leading order lagrangian describes the interaction of baryons with light meson vector- (V_μ) and axial-currents (A_μ)

$$\mathcal{L} = tr\,\bar{B}_v iv{\cdot}DB_v + D\,tr\,\bar{B}_v\gamma^\mu\gamma_5\{A_\mu, B_v\} + F\,tr\,\bar{B}_v\gamma^\mu\gamma_5[A_\mu, B_v] + O\left(\frac{1}{m_B}\right) + \mathcal{L}_\pi\,,$$

$$(2)$$

where
$$B_v(x) = \frac{1 + \not v}{2}\,B(x)\,e^{im_B v\cdot x} \qquad F + D = g_A\,, \qquad (3)$$

with the field $B_v(x)$ containing the residual momentum dependence, after the large factor $m_B v$ has been factored out. The expansion of HBChPT is therefore a double expansion in $1/m_B$ and in $1/\Lambda_\chi$. The first sets the scale of the breaking of heavy quark spin symmetry, the second sets the scale of the breaking of chiral symmetry. An analogous merging gave rise to the description of hadrons with a heavy quark, the EFT known as Heavy-Light ChPT (HLChPT) [11], which describes the strong interactions of D, D* and B, B* mesons with pions.

3.2 Chiral perturbation theory and lattice QCD

Field theories can be formulated on a euclidean world-grid, where space and time are discretized and the unit distance, the lattice spacing, acts as the ultraviolet regulator of the theory. The euclidean formulation allows for a statistical intepretation of the path integral and its treatment with Monte Carlo methods (see ref. [18] for a review). Typical lattice simulations of QCD are performed on a hypercube with volume $L^3 \times L_t$, with spatial extension $L = Na$, temporal extension $L_t = N_t a$ and lattice spacing a (in some cases a different lattice spacing $a_s \neq a_t$ might be conveniently chosen). For an introduction to lattice field theory and lattice QCD see e.g. [19]. The lattice formulation allows for a *first principle* description of a theory, both in the strong-coupling (non perturbative) and weak-coupling (perturbative) regimes. Ideally $1/L \ll m_\pi \ll \Lambda_\chi \ll 1/a$ guarantees that pions freely move in the lattice box, i.e. their Compton wavelenght is much smaller than L, and they do not feel the discretization of spacetime. The goal is to be as near as possible to the real world limits $L \to \infty$ (the infinite volume limit), $a \to 0$ (the continuum limit), and $m_{u,d} \sim m_{u,d}^{phys}$ (the chiral limit for $m_{u,d}^{phys} \simeq 0$). Typical magnitudes for simulations up to date are $L \sim 2 \div 4\ fm$, $a \leq 0.1\ fm$, and $m_{u,d} \leq m_s/2$. Last

years have seen an enormous improvement, with simulations at lattice spacings down to $a \sim 0.05\ fm$ and quark masses as small as $m_{u,d} \sim m_s/8$.

The Symanzik action [20] was the first example of EFT used to guide a lattice calculation, in this case to perform the extrapolation to the continuum limit. The generalization of this approach is an EFT description that guides the extrapolation to all limits, the infinite volume, the continuum and the chiral limit. During many years the quenched approximation of QCD (QQCD), where the fermionic determinant in the path integral is set to a constant, was a forced choice for lattice calculations[6]. On the way to restore the original content of QCD, one can formulate a partially quenched (PQQCD) version of it and the corresponding (partially) quenched ChPT [21, 22], where sea quarks are distinguished from valence quarks and added at will to the theory content. Valence quarks are quenched, while sea quarks are dynamical. The QCD point is recovered at $N_{sea} = N_{valence}$ and $m_{sea} = m_{valence}$. The symmetry group is the graded extension [21] of the chiral group $SU(N) \times SU(N)$ for N flavours: $SU(N|N) \times SU(N|N)$, with N valence and ghost quarks in the quenched case, and $SU(N+K|N) \times SU(N+K|N)$, for K sea quarks and N valence and ghost quarks, in the partially quenched extension. The construction of (P)QChPT, initiated a stream of results which quickly clarified how the approximation affects observables and their volume dependence, using symmetry arguments, the non-unitarity of the quenched theory, and group theory considerations [23]. Further experience in the EFT approach *à la* Symanzik allowed to guide simulations towards new regimes of masses and volumes, from the usual p-regime to the ε-regime when approaching the chiral limit: the original theoretical formulation [24] was riproposed [25] in a lattice context. From the p-regime, with moderately large volumes and masses, $m_\pi L, m_\pi L_t \gg 1$, $2\pi/L \ll \Lambda_\chi \ll 1/a$ and p/Λ_χ small, one enters the ε-regime while lowering the quark masses, where $m_\pi L, m_\pi L_t \sim \varepsilon \ll 1$. Here the zero modes [26] of the Dirac operator must be resummed: $m_q \langle \bar{q}q \rangle L^3 L_4 \leq O(1)$.

Nowadays, ChPT formulations match every possible lattice strategy, mainly depending on the way fermions are included in the lattice action. The physical prediction is unique, but not the way a specific lattice formulation extrapolates to the chiral and continuum limits.

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[6] The fermionic operator is a large sparse matrix of $spins \times colours \times space \times time$, that renders the exact calculation computationally very expensive.

4   Hot Phenomenology

Hot topics of today phenomenology are those providing a powerful probe of physics beyond the standard model. Restricting to (almost light) hadron phenomenology brings me to mention topics as rare kaon decays, which provide tests of the unitarity of the CKM matrix, and charm physics. The following short sections are meant to recall a few important aspects, while for an extended analysis I refer the reader to the existing literature.

4.1   Rare decays and CKM unitarity

The semileptonic decay Kl_3 is a gold plated kaon decay. It can provide [27] a precise test of lepton universality, a determination of the amount of SU(2) breaking, through mass ratios, and the amount of SU(3) breaking, through the determination of the CKM matrix element V_{us}. The rare semileptonic processes $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are crucial channels to probe new physics contributions. For the latter processes, the accurate knowledge of the charm mass is crucial. All standard model contributions to these processes are being calculated with incresing accuracy and with use of ChPT for long distance contributions. For an updated overview, visit the Kaon 2007 website [28]. Finally, the radiative decays $K \rightarrow \pi \gamma \gamma, \pi \pi \gamma$ can further probe the range of validity of ChPT and long distance dynamics.

4.2   Charm physics

The physics of charm is as rich as difficult to decipher. The charm is not heavy enough M_c is not $\gg \Lambda_\chi$ to use the heavy quark expansion with sufficiently high predictive power, and it is not light enough $M_c \simeq \Lambda_\chi$ to use the chiral expansion. However, it is more relativistic than the bottom quark, hence its lattice formulation is affected by smaller discretization errors. We need $m_c \ll 1/a$ on the lattice, and it is now easy to get $m_c a \sim 1/2$ [29]. What is further needed? Two points are worth to be mentioned: i) a more accurate determination of the charm mass (to the percent level), and ii) the prediction of the strong interaction phases of D-meson decays which probe CP violation and are indirect probes of physics beyond the standard model.

5 Conclusive thoughts

During the last two decades we have reshaped our view of quantum field theories. Effective field theories are at the foundation of modern quantum field theory, and the effective field theory of low energy QCD has significantly contributed to this view. Where is the future of EFTs? They will probably remain for long the bread and butter of field theoretical approaches to many phenomena, not only in particle physics, but widely used in condensed matter physics. There are clear places in particle physics where the formulation of an effective field theory description still needs to be fruitfully improved. This is the case for the prediction of the electric dipole moments, tiny observables measured at very high precision low energy experiments [30]. Can one think of a new hybrid EFT formalism to efficiently describe strong interactions in charm decays? or the yet unexplored intermediate regime of baryon densities in neutron stars? One important role of EFT is undeniably the one of uncovering the possible connection of (super)gravity theories to a four-dimensional universe.

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